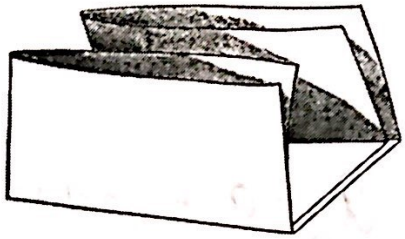
Lesson 5: Geometric Sequences

Task

If you folded a piece of paper in half 50 times how thick would it be?



The Answer:

Number of folds	0	1	2	3	4	5	6
~~Times as thick~~	1	2	4	8	16	32	64
LAYERS	2^0	2^1	2^2	2^3	2^4	2^5	2^6

← THE VALUE OF n IS IN THE EXPONENT

What is the pattern:

EACH TERM IS MULTIPLIED BY 2

You try with your piece of paper.

The world record for folding a paper in half is 13

A piece of paper is approximately:

254 pieces of paper = _____

So _____ divided by _____ = _____

There are ____ inches in ___ foot

So _____ divided by _____ = _____

There are _____ feet in ___ mile

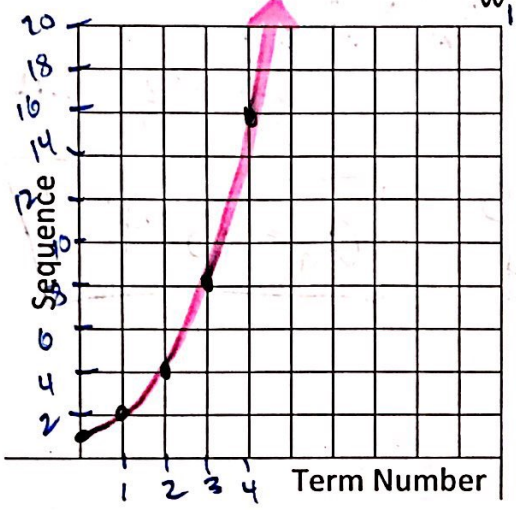
So _____ divided by _____ = _____

What about this **sequence from above:**?

(1) 2, 4, 8, 16, 32, 64

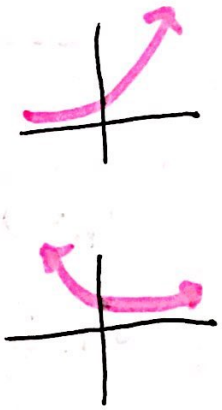
a_0 ↑ a_1 ↑ a_3

Graph it to see a pattern



EXPONENTIAL FUNCTION

2^n



What is the 3rd **term** of the sequence?

$a_3 = 8$

What is the 10th term of the sequence?

$2^{10} = 1{,}024 = a_{10}$

Part 2: Geometric Sequence

Geometric Sequence:

A pattern of terms where each term after the first is found by MULTIPLYING the previous term by a non-zero number called the common ratio, r.

Term (a_1 and a_n):

a_1: FIRST TERM IN SEQUENCE

a_n: VALUE OF TERM YOU GET AFTER SUBSTITUTING IN n.

n: TERM # YOU WANT TO FIND

Common ratio: # you are multiplying each term by

$r = \frac{\text{2nd term value}}{\text{1st term value}}$

Explicit rule:

$a_n = a_1 \cdot r^{n-1}$

Recursive rule:

Part 3 Patterns and Sequences Examples

4. Given the table

n	1	2	3	4
a_n	2	6	18	54

a. What is the common ratio?

$r = 3$

b. Write the explicit rule:

$a_n = a_1 \cdot r^{n-1}$

$a_n = 2 \cdot 3^{n-1}$

c. Write the recursive rule:

d. Find the 8th term.

$a_8 = 2 \cdot 3^{8-1}$

$= 2 \cdot 3^7$

$= 2 \cdot 2187$

$a_8 = 4374$

5. Given the table: ÷ −4

n	2	3	4	5
a_n	20	−80	320	−1,280

(n = 1: −5; × −4)

a. What is the common ratio?

$r = \frac{-80}{20} = -4$

b. Write the explicit rule:

$a_1 = -5$

$r = -4$

$a_n = -5 \cdot -4^{n-1}$

c. Write the recursive rule:

d. Find the 12th term.

$a_{12} = -5 \cdot -4^{12-1}$

$= -5(-4^{11})$

$= (-5) \cdot -4194304$

$a_{12} = 20{,}971{,}520$

6. Consider the sequence 81, 27, 9, 3, …

a. What is the common ratio?

$= \frac{27}{81} \div 9 = \frac{3}{9}$

$r = \frac{1}{3}$

b. Write the explicit rule:

$a_n = a_1 \cdot r^{n-1}$

$a_n = 81 \cdot \frac{1}{3}^{n-1}$

~~c. Write the recursive rule:~~

$a_9 = 81 \cdot \frac{1}{3}^{9-1}$

$81\left(\frac{1}{3}\right)^8$

$a_9 \approx 0.123$

d. Find the 9th term.

$81 \cdot \frac{1}{6561} = \frac{81 \div 81}{6561 \div 81}$

$a_9 = \frac{1}{81}$

7. Consider the sequence −24, 12, −6, 3, …

a. What is the common ratio?

$r = \frac{12}{-24} = -\frac{1}{2}$

b. Write the explicit rule:

$r = -\frac{1}{2}$, $a_1 = -24$

$a_n = a_1 \cdot r^{n-1}$

$= -24 \cdot \left(-\frac{1}{2}\right)^{n-1}$

~~c. Write the recursive rule:~~

$a_{11} = -24\left(-\frac{1}{2}\right)^{11-1}$

$= -24\left(-\frac{1}{2}\right)^{10}$

$= -24\left(\frac{1}{2^{10}}\right)$

d. Find the 11th term.

$a_{11} = -24\left(\frac{1}{1024}\right)$

$a_{11} = \frac{-24}{1024} \div 8 = \frac{-3}{128}$

Ticket out the door:

Create your own geometric sequence. What makes it a geometric sequence?