

Exploring Quadratic Function in Intercept Form.

What could you do with a function in this form?

$f(x) = 2(x+1)(x-3)$ Solve it: $x = -1$ ←
 $x+1=0$ $x-3=0$ $x=3$

X-INTERCEPTS = SOLUTIONS = ROOTS

Write the quadratic above in standard form.

$f(x) = 2(x+1)(x-3)$ → $x+1$

x	$x^2 + x$
-3	$-3x - 3$

 $f(x) = 2(x^2 - 2x - 3)$
 $f(x) = 2x^2 - 4x - 6$

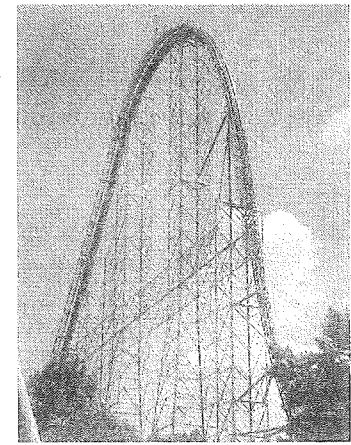
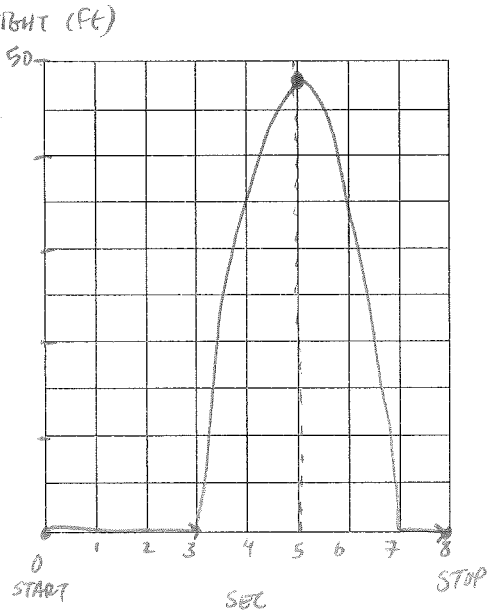
THESE EQUATIONS ARE EQUIVALENT; WHEN THE STANDARD FORM IS FACTORED AND SOLVED, IT LEADS TO THE SOLUTIONS WHICH ARE THE X-INTERCEPTS OF THE GRAPH. (ALSO CALLED ROOTS)

A rollercoaster travels at a constant rate for 8 seconds. It begins by traveling horizontally; after three seconds it begins inclining until it reaches its peak and then drops down. When the coaster begins traveling horizontally again, seven seconds have passed. It then travels horizontally for 1 second before the rate of the coaster changes. The engineer of the rollercoaster tells you that the maximum the coaster goes is 48 feet.

Draw a "rough" graph of what you know so far about this graph, your focus primarily being the parabola shape that has been formed.

Info that we know/can deduct:

- RIDE DESCRIPTION IS 8 SEC
- NO INCLINE FOR FIRST 3 SEC; THEN BEGINS INCLINE - FLATTENS AGAIN @ 7 SEC
- MAX: 48 FT
- MAX WOULD HAPPEN HALF-WAY BETWEEN 3 SEC & 7 SEC: 5 SEC



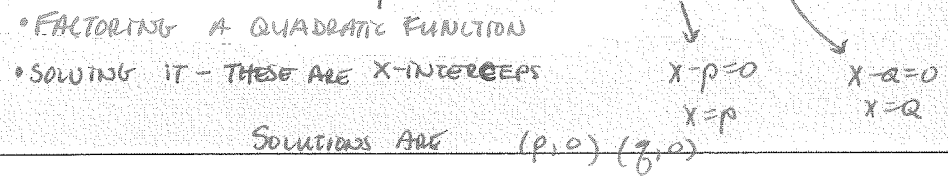
Find the function

Vertex: (5,48)
 X-INT: (3,0) (7,0)

The X-INTS came from the following binomials:

$x=3$ $x=7$
 ↓ so
 $x-3=0$ $x-7=0$

Intercept form of a quadratic function: is $f(x) = a(x-p)(x-q)$ where the intercepts are found by...



Exploring the graph of a Quadratic Function in intercept form.

Graph: $g(x) = (x + 2)(x - 4)$

$a=1$

Steps to graph a quadratic function in Intercept

Form: $f(x) = a(x - p)(x - q)$:

1. DIRECTION OF OPENING USING a

$a=1$ $+a$
↑ opens up

2. FIND SOLUTIONS / X-INTERCEPTS USING FACTORED FORM: (SET BINOMIALS EQUAL TO ZERO)

$x+2=0$
 $-2 -2$

$x=-2$
 $(-2,0)$

$x-4=0$
 $+4 +4$

$x=4$
 $(4,0)$

• PLOT

3. FIND AXIS OF SYMMETRY: COUNT HALF-WAY BETWEEN X-INTERCEPTS

• PLOT IT

• IDENTIFY AS $x = \underline{\hspace{2cm}}$

• REMEMBER, THIS IS X-COORD OF VERTEX

4.

FIND y-COORDINATE OF VERTEX:

• SUBSTITUTE X-COORD INTO INTERCEPT FORM

5. FIND y-INTERCEPT: $x=0$

• MAKE A COPY

6. CONNECT POINTS TO MAKE PARABOLA

Direction of opening: opens up

Vertex: $(1, -9)$

Maximum/Minimum: $y = -9$

Axis of Symmetry: $x = 1$

y-intercept: $(0, -8)$

x-intercept(s): $(-2, 0)$ $(4, 0)$

Domain: \mathbb{R}

Range: $\{y \mid y \geq -9\}$

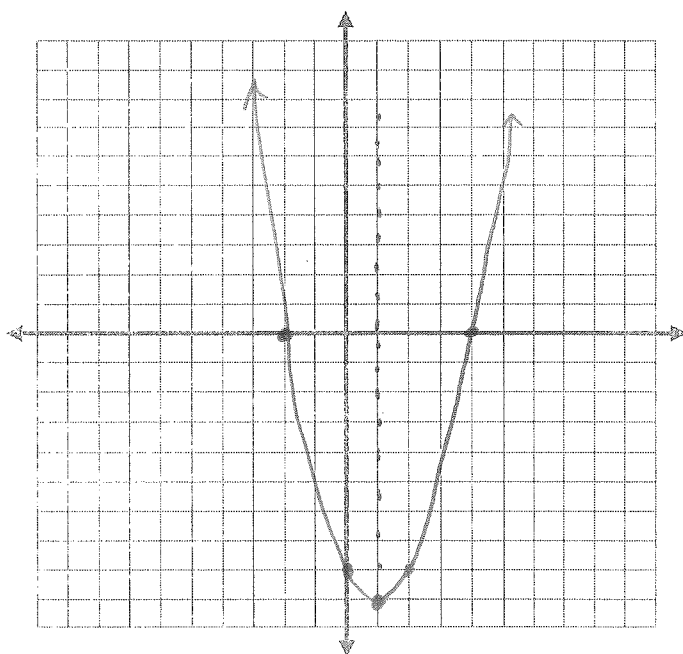
x	g(x)
2	-8
-2	0
1	-9
4	0
0	-8

Vertex: $(1, -9)$

$g(1) = (1+2)(1-4)$
 $= (3)(-3)$
 $= -9$

y-int: $(0, -8)$

$g(0) = (0+2)(0-4)$
 $= (2)(-4)$
 $= -8$



Examples: Find all the parts and graph the quadratic function in intercept form.

$$f(x) = 2(x-1)(x+5) \quad a=2$$

x	f(x)
0	-10
1	0
-2	-18
-5	0
-4	-10

Direction of opening: opens up

Vertex: (-2, -18)

Maximum/Minimum: $y = -18$

Axis of Symmetry: $x = -2$

y-intercept: (0, -10)

X-INT:

$$\begin{array}{l} x-1=0 \quad x+5=0 \\ x=1 \quad x=-5 \end{array}$$

x-intercept(s): (1, 0) (-5, 0)

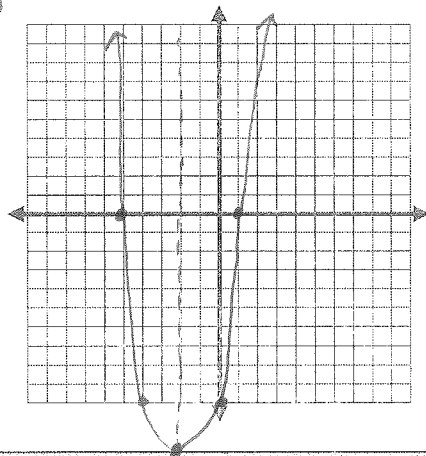
Domain: \mathbb{R} Range: $\{y | y \geq -18\}$

VERTEX:

$$\begin{aligned} f(-2) &= 2(-2-1)(-2+5) \\ &= 2(-3)(3) \\ &= -18 \end{aligned}$$

y-INT: $x=0$

$$\begin{aligned} f(0) &= 2(0-1)(0+5) \\ &= 2(-1)(5) \\ &= -10 \end{aligned}$$



$$f(x) = -\frac{1}{2}(x-1)(x+7) \quad a = -\frac{1}{2}$$

x	f(x)
-6	3.5
-7	0
-3	8
1	0
0	3.5

Direction of opening: opens down

Vertex: (-3, 8)

Maximum/Minimum: $y = 8$

Axis of Symmetry: $x = -3$

y-intercept: (0, 3.5)

X-INT:

$$\begin{array}{l} x-1=0 \quad x+7=0 \\ x=1 \quad x=-7 \end{array}$$

x-intercept(s): (1, 0) (-7, 0)

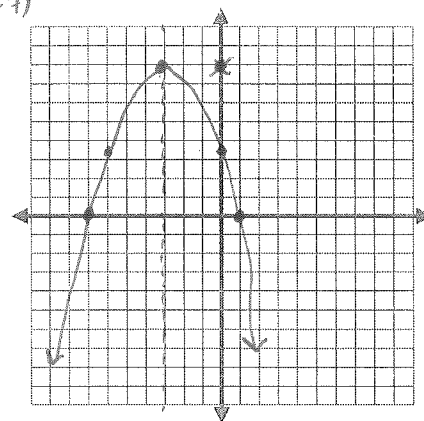
Domain: \mathbb{R} Range: $\{y | y \leq 8\}$

Vertex:

$$\begin{aligned} f(-3) &= -\frac{1}{2}(-3-1)(-3+7) \\ &= -\frac{1}{2}(-4)(4) \\ &= -\frac{1}{2}(-16) \\ &= 8 \end{aligned}$$

y-INT: $x=0$

$$\begin{aligned} f(0) &= -\frac{1}{2}(0-1)(0+7) \\ &= -\frac{1}{2}(-1)(7) \\ &= \frac{1}{2}(7) \\ &= 3.5 \end{aligned}$$



Examples of Changing from Intercept Form to Standard Form

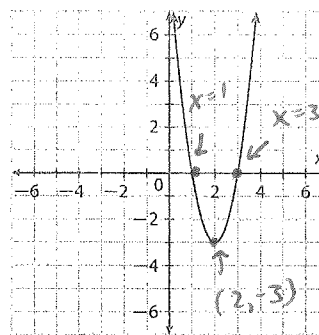
$$f(x) = -\frac{1}{2}(x+6)(x-2)$$

$$\begin{array}{r} (x+b) \\ x \quad x^2 \quad | \quad 6x \\ -2 \quad -2x \quad | \quad -12 \end{array}$$

$$f(x) = -\frac{1}{2}(x^2 + 4x - 12)$$

$$f(x) = -\frac{1}{2}x^2 - 2x + 6$$

Write the equation in intercept form and then change it to standard form.



$$\begin{aligned} x=1 \quad x=3 \\ \downarrow \quad \downarrow \\ a(x-1)(x-3) = f(x) \\ \text{use } (2, -3) \text{ to solve for } a. \\ a(2-1)(2-3) = -3 \\ a(1)(-1) = -3 \\ -1a = -3 \\ -1 \cdot a = -3 \\ a = 3 \end{aligned}$$

$$\begin{array}{r} x \quad -1 \\ x \quad x^2 \quad | \quad -1x \\ -3 \quad -3x \quad | \quad +3 \end{array}$$

$$\begin{aligned} f(x) &= 3(x-1)(x-3) \\ &= 3(x^2 - 4x + 3) \end{aligned}$$

$$f(x) = 3x^2 - 12x + 9$$