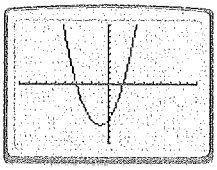
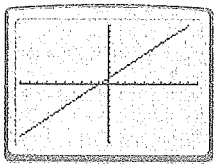
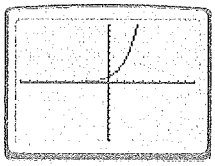
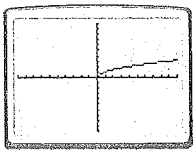
Algebra 1: Unit 7, lesson 4 Notes: Interpreting Vertex and Standard Form Name: Key

Identifying quadratic functions from their graphs and equations

Is the following a quadratic function: How do you know?

$f(x)=2x^2-5$ yes: x^2	$g(x)=\sqrt{x}$ No - No x^2	$h(x)=2x+4$ No - LINEAR
yes - PARABOLA 	 No - LINEAR	 No - NOT A PARABOLA
$h(x)=-2(x-3)^2$ yes - VERTEX FORM	 No	$f(x)=2^{x-3}$ No

Changing from Vertex Form to Standard Form

Rewrite a quadratic function from vertex form: $y=a(x-h)^2+k$, to standard form: $y=ax^2+Bx+c$.

$y=2(x+4)^2-10$

	x	$+4$
x	x^2	$+4x$
$+4$	$+4x$	$+16$

$y=2(x^2+8x+16)-10$

$=2x^2+16x+32-10$

$y=2x^2+16x+22$

Steps

1. Expand THE BINOMIAL
   - BOX
   - DISTRIBUTE
2. DISTRIBUTE a
3. COMBINE LIKE-TERMS.

Examples of Changing from Vertex Form to Standard Form

$y=3(x-2)^2+4$

	x	-2
x	x^2	$-2x$
-2	$-2x$	$+4$

$y=3(x^2-4x+4)+4$

$=3x^2-12x+12+4$

$y=3x^2-12x+16$

$f(x)=\frac{1}{2}(x+6)^2-8$

	x	$+6$
x	x^2	$+6x$
$+6$	$+6x$	$+36$

$=\frac{1}{2}(x^2+12x+36)-8$

$=\frac{1}{2}x^2+6x+18-8$

$f(x)=\frac{1}{2}x^2+6x+10$

Algebra 1: Unit 7, lesson 4 Notes: Interpreting Vertex and Standard Form Name:

Writing a Quadratic Function Given a Table of Values in Vertex Form

x	y
−6	9
−4	1
−3	0
−2	1
0	9

Vertex: $(-3, 0)$

$y = a(x - -3)^2 + 0$

$y = a(x + 3)^2 + 0$

$y = a(x + 3)^2 + 0$

$(0, 9)$
x y

$9 = a(0 + 3)^2 + 0$

$9 = a(3^2) + 0$

$\frac{9}{9} = \frac{9a}{9}$

$1 = a$

$y = 1(x + 3)^2 + 0$

Steps

1. Identify the VERTEX.
   - THE VERTEX IS THE ORDERED PAIR WITH A UNIQUE y-COORDINATE
2. Substitute VERTEX COORDINATES for (h, k) in VERTEX FORM.
   $y = a(x - h)^2 + k$
3. Substitute ANY ORDERED PAIR from the table in to the equation to find a.
   - Solve for a.
4. Write the EQUATION of the quadratic function.
   USING a, h, and k in VERTEX FORM.

Examples of Writing a Quadratic Function Given a Table of Values in Vertex Form

x	y
0	13
−1	1
−2	−3
−3	1
−4	13

VERTEX: $(-2, -3)$ h, k

$y = a(x - h)^2 + k$

$y = a(x - -2)^2 + -3$

$y = a(x + 2)^2 - 3$

$(0, 13)$

$13 = a(0 + 2)^2 - 3$

$13 = a(2^2) - 3$

$+3 \quad +3$

$\frac{16}{4} = \frac{4a}{4}$

$4 = a$

$y = 4(x + 2)^2 - 3$

x	y
−1	59
1	11
2	5
3	11
5	59

VERTEX: $(2, 5)$ h, k

$y = a(x - h)^2 + k$

$y = a(x - 2)^2 + 5$

$(1, 11)$

$11 = a(1 - 2)^2 + 5$

$-5 \quad -5$

$6 = a(-1^2)$

$6 = 1a$

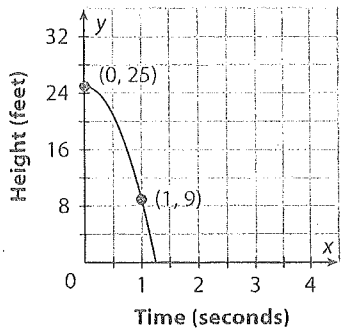
$6 = a$

$y = 6(x - 2)^2 + 5$

Algebra 1: Unit 7, lesson 4 Notes: Interpreting Vertex and Standard Form Name:

Writing a Quadratic Function Given a Graph

A house painter standing on a ladder drops a paintbrush, which falls to the ground. The paintbrush's height above the ground (in feet) is given by a function of the form $f(t) = a(t-h)^2 + k$ where t is the time (in seconds) after the paintbrush is dropped.



WRITE THE QUADRATIC FUNCTION (USING THE GRAPH) WHICH REPRESENTS THE PAINTBRUSHES HEIGHT ABOVE THE GROUND AT ANY TIME.

VERTEX: (0, 25) → h, k

POINT: (1, 9)

$f(t) = a(t-h)^2 + k$

$f(t) = a(t-0)^2 + 25$

$9 = a(1-0)^2 + 25$

$-25 \quad -25$

$-16 = a(1^2)$

$-16 = 1a$

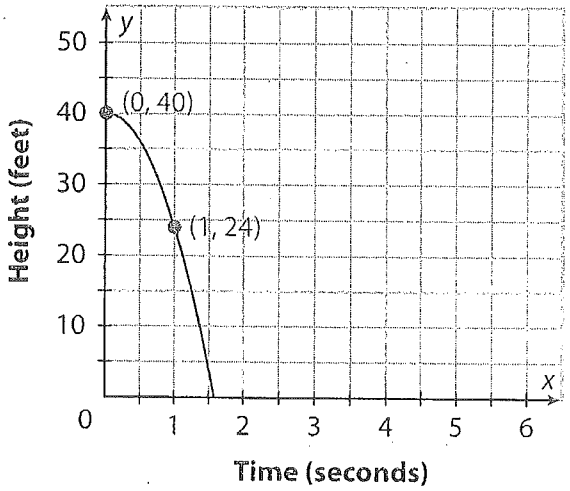
$f(t) = -16(t-0)^2 + 25$

Steps

1. USE COORDINATES OF VERTEX, (h, k) TO SUBSTITUTE INTO VERTEX FORM
2. SUBSTITUTE IN COORDINATES OF ANOTHER POINT ON THE GRAPH; SOLVE FOR a.

Example of Writing a Quadratic Function Given a Graph

A rock knocked off a cliff into the water far below. The falling rock's height above the water (in feet) is given by a function of the form $f(t) = a(t-h)^2 + k$ where t is the time (in seconds) after the rock begins to fall.



VERTEX: (0, 40) → h, k

POINT: (1, 24) → t, f(t)

$f(t) = a(t-0)^2 + 40$

$24 = a(1-0)^2 + 40$

$-40 \quad -40$

$-16 = a(1^2)$

$-16 = 1a$

$a = -16$

$y = -16(t-0)^2 + 40$

USE YOUR FUNCTION TO ESTIMATE THE HEIGHT OF THE ROCK AFTER 0.5 SEC.

$y = -16(0.5-0)^2 + 40$

$f(0.5) = -16(0.5)^2 + 40$

$= -16(0.25) + 40$

$= -4 + 40$

$f(0.5) = 36$

USE YOUR FUNCTION TO ESTIMATE HOW LONG IT TAKES THE ROCK TO HIT THE WATER.

* HIT THE WATER MEANS $f(x) = 0$

$$0 = -16(t-0)^2 + 40$$

$$\frac{-40}{-16} = \frac{-16(t-0)^2}{-16}$$

$$2.5 = (t-0)^2$$

$1.58 \approx t$
SECONDS