Algebra 1: Unit 7, lesson 2 Notes: Graphing Quadratic Functions: Standard Form Name: KEY

Part 1: Revisit $f(x) = ax^2$

What do we know about "*a*"

- a affects the wideness of a parabola
  - $0<a<1$: wider
  - $a>1$: narrower
- a affects if the parabola opens up or down
  - $+a$: opens up
  - $-a$: opens down

How do we know if the parabola has a maximum or a minimum?

- It depends on if the parabola opens Up or Down (a^+, minimum) (a^-, maximum)

$$f(x) = ax^2 + bx + c$$

Where have you seen this function before?

- FACTORING
- QUADRATIC FORMULA

Any ideas of what "*b*" and "*c*" might do to change the graph?

- Maybe it moves the graph from the origin - right vs left, up vs down

Let's look at a problem with only "*b*" and the shape of our parabola looking like shots from our Basketball Task. (*a* = -1) ⇦ OPENS DOWN

x	-2	-1	0	1	2	3
$f(x) = -x^2 + 4x$	-12	-5	0	3	4	3

$f(-2) = -1\cdot(-2^2) + 4(-2)$
$= -4 - 8$
$= -12$

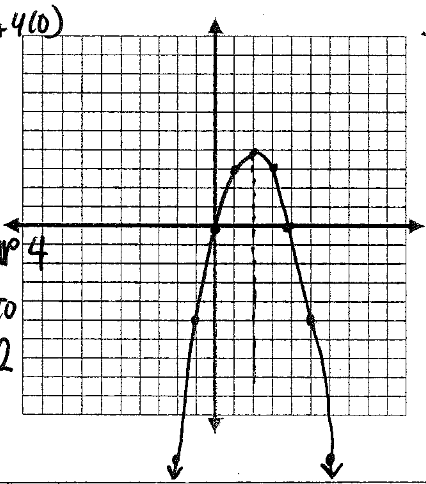
$f(1) = -1\cdot(1^2) + 4(1)$
$= -1 + 4$
$= 3$

$f(-1) = -1\cdot(-1^2) + 4(-1)$
$= -1 - 4$
$= -5$

$f(2) = -1(2^2) + 4(2)$
$= -4 + 8$
$= 4$

$f(0) = 0^2 + 4(0)$

$f(3) =$
$-1\cdot 3^2 + 4(3)$
$-9 + 12$



VERTEX
- MOVED UP 4
- MOVED TO RIGHT 2

Let's look at a problem with only "*c*" and the shape of our parabola looking like shots from our Basketball Task. (*a* = -1)

x	-2	-1	0	1	2
$f(x) = -x^2 + 4$	0	3	4	3	0

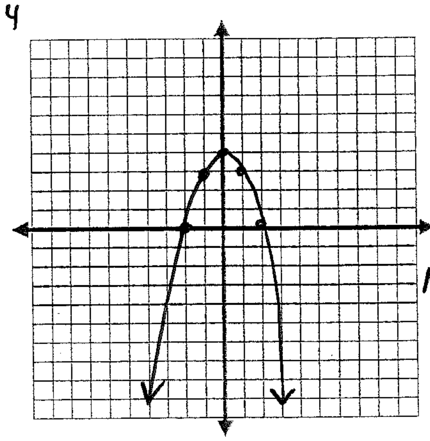
$f(-2) = -1\cdot(-2^2) + 4$
$= -4 + 4$
$= 0$

$f(1) = -1\cdot 1^2 + 4$
$= -1 + 4$
$= 3$

$f(-1) = -1\cdot(-1^2) + 4$
$= -1 + 4$
$= 3$

$f(2) = -1\cdot 2^2 + 4$
$= -4 + 4$
$= 0$

$f(0) = -0^2 + 4$
$= 4$



VERTEX MOVED UP 4; NO SIDE TO SIDE

What findings do you see:

Lets look more at "c"

x	-2	-1	0	1	+2
$f(x) = x^2 + 4x + 8$	4	5	8	13	20

$f(-2) = (-2)^2 + 4(-2) + 8$
$= 4 - 8 + 8$

$f(-1) = (-1)^2 + 4(-1) + 8$
$= 1 - 4 + 8$
$= -3 + 8$

$f(0) = 0^2 + 4(0) + 8$
$f(0) = 8$

$f(1) = 1^2 + 4(1) + 8$
$= 1 + 4 + 8$
$= 13$

$f(2) = 2^2 + 4(2) + 8$
$= 4 + 8 + 8$
$= 20$

x	-2	-1	0	1	+2
$f(x) = -x^2 - 6x - 7$	1	-2	-7	-14	-23

$f(-2) = -1 \cdot (-2^2) - 6(-2) - 7$
$= -4 + 12 - 7$
$= 1$

$f(-1) = -1 \cdot (-1^2) - 6(-1) - 7$
$= -1 + 6 - 7$
$= -2$

$f(0) = 0^2 - 6(0) - 7$
$= -7$

$f(1) = -1^2 - 6(1) - 7$
$= -1 - 6 - 7$
$= -14$

$f(2) = -1 \cdot 2^2 - 6(2) - 7$
$= -4 - 12 - 7$
$= -23$

What does the "c" stand for:

Thinking back to our Basketball Task what might "c" be in those problems?

Basketball Task Day 2 Part 1 – see Task Handout and transparency graph paper

Part 2: Exploring the graph of a Quadratic Function in Standard form:

$$f(x) = ax^2 + Bx + C$$

$f(x) = x^2 + 4x - 5$

$a = 1$
$B = 4$
$C = -5$

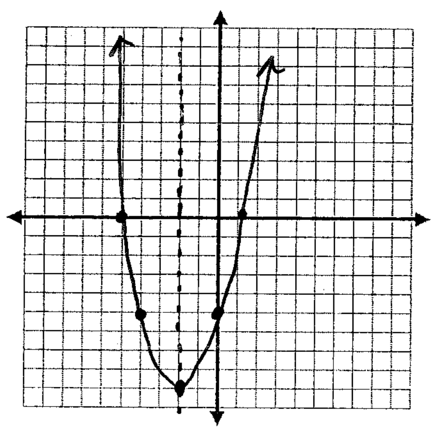
$x = \frac{-4}{2(1)} = -2$

$f(-2) = (-2^2) + 4(-2) - 5$
$= 4 - 8 - 5$
$= -9$

$x^2 + 4x - 5$

$(x + 5)(x - 1)$

$x + 5 = 0$ $\quad$ $x - 1 = 0$
$x = -5$ $\quad$ $x = 1$



Steps to find Parts of a Quadratic Function $f(x) = ax^2 + bx + c$:

1. IDENTIFY a, b, c
2. CALCULATE AXIS OF SYMMETRY
   $X = \frac{-B}{2A}$ → PLOT.
   THIS IS ALSO X-COORD. OF VERTEX
3. VERTEX: SUBSTITUTE AXIS OF SYMMETRY INTO FUNCTION
   - EVALUATE.
   - THIS IS y-COORD OF VERTEX
4. - PLOT VERTEX
   C IS y-INTERCEPT. $(0, C)$
   PLOT.
   MAKE COPY OVER AXIS OF SYMMETRY.
5. FIND X-INTERCEPTS BY FACTORING. ORIGINAL FUNCTION - SOLVE.
   $(x_1, 0)$ $\quad$ $(x_2, 0)$
6. CONNECT POINTS.

Parts of a Quadratic Function:

	x	y	
	-5	0	
	-4	-5	copy
V	-2	-9	
	0	-5	y-INT
	~~-5~~ 1	0	

Direction of opening: opens up

Vertex: $(-2, -9)$

Maximum/Minimum: $y = -9$

Axis of Symmetry: $x = -2$

y-intercept: $(0, -5)$ $\quad$ x-intercept(s): $(-5, 0)$ $(1, 0)$

Domain: $\mathbb{R}$

Range: $\{y \mid y \geq -9\}$

Algebra 1: Unit 7, lesson 2 Notes: Graphing Quadratic Functions: Standard Form Name:

Examples: Find all the parts and graph the quadratic function in standard form.

Ex 1: $f(x) = -\frac{1}{2}x^2 + 2x + 6$

$a = -\frac{1}{2}$

$B = 2$

$C = 6$

$x = \frac{-B}{2A} \rightarrow x = \frac{-2}{2(-\frac{1}{2})} \Rightarrow \frac{-2}{-1} \rightarrow x = 2$

$f(-1) = -\frac{1}{2}(2^2) + 2(2) + 6$

$= -\frac{1}{2}(4) + 4 + 6$

$= -2 + 4 + 6$

$= 8$

FACTOR BY BOTTOMS UP!

$-\frac{1}{2}x^2 + 2x + 6$

$x^2 + 2x - 3$

$(x - \frac{1}{-\frac{1}{2}})(x + \frac{3}{-\frac{1}{2}})$

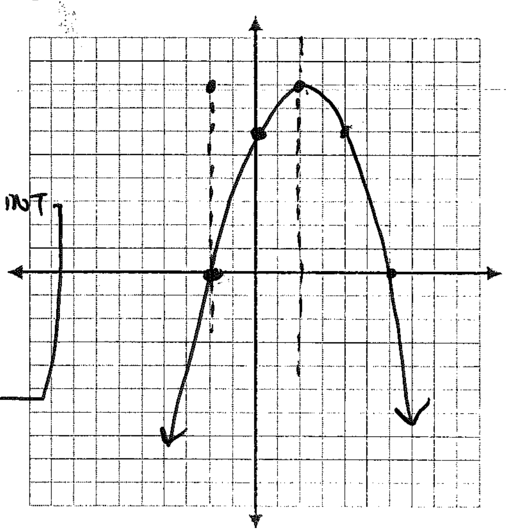
$-\frac{1}{2}x - 1 = 0$ $\quad$ $-\frac{1}{2}x + 3 = 0$

$-2 \cdot -\frac{1}{2}x = 1 \cdot -2$ $\quad$ $-2 \cdot -\frac{1}{2}x = -3 \cdot -2$

$x = -2$ $\quad$ $x = 6$

$(-2, 0)$ $\quad$ $(6, 0)$

x	y
-2	0
0	6 ← y INT
+2	8
4	6
6	0



Parts of a Quadratic Function:

Direction of opening: opens down

Vertex: $(2, 8)$

Maximum/Minimum: $y = 8$

Axis of Symmetry: $x = 2$

y-intercept: $(0, 6)$

x-intercept(s): $(-2, 0)$ $(6, 0)$

Domain: $\mathbb{R}$

Range: $\{y \mid y \leq 8\}$

$f(x) = 3x^2 + 6x + 1$

$a = 3$

$B = 6$

$C = 1$

$x = \frac{-B}{2A}$

$x = \frac{-6}{2(3)} = -1$

$f(-1) = 3(-1^2) + 6(-1) + 1$

$= 3 - 6 + 1$

$= -3 + 1$

$= -2$

FACTOR BY BOTTOMS UP: $3x^2 + 6x + 1 = 0$

$x^2 + 6x + 3 = 0$

does not factor

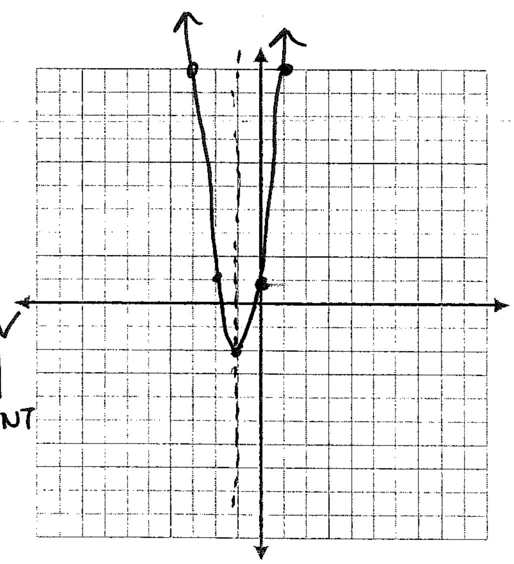
CHOOSE ANOTHER x IF IT CAN'T FACTOR

$f(1) = 3(1^2) + 6(1) + 1$

$= 3 + 6 + 1$

$= 10$

x	y
-3	10
-2	1
-1	-2
0	1 y INT
1	10



Parts of a Quadratic Function:

Direction of opening: Up

Vertex: $(-1, -2)$

Maximum/Minimum: $y = -2$

Axis of Symmetry: $x = -1$

look where the curve crosses x-axis: guess!

y-intercept: $(0, 1)$

x-intercept(s): (ABOUT) $(-\frac{1}{4}, 0)$ and $(-1\frac{3}{4}, 0)$

Domain: $\mathbb{R}$

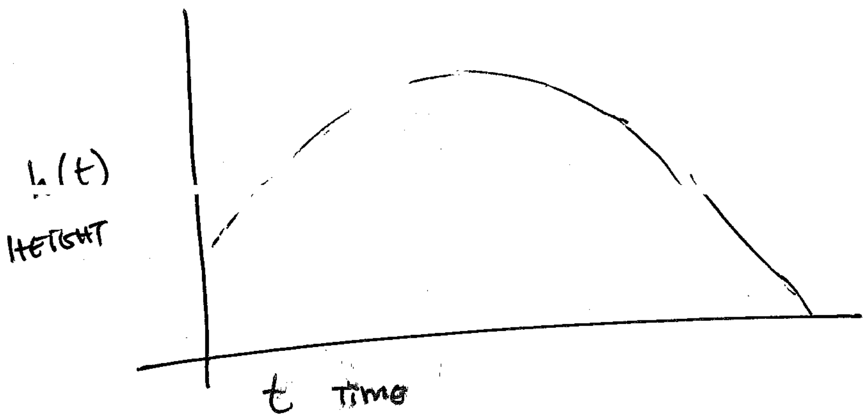
Range: $\{y \mid y \geq -2\}$

Exploring the equation of a quadratic function in standard form.

Tim hits a softball. The function $h(x) = -14t^2 + 56t + 3$ describes the height (in feet) of the softball, and *t* is th time (in seconds).

a) Draw a rough graph of what this graph might look like.

$a = -14$
$B = 56$
$C = 3$



b) Does the graph have a maximum or minimum? What is it? Explain in the context of the problem.

THE VERTEX — THE PEAK OF THE BALL

$(2, 59)$
2 sec — 59 feet

$X = \frac{-56}{2(-14)} = \frac{56}{28} = 2 \text{ sec}$

$h(2) = -14(2^2) + 56(2) + 3$
$= -14(4) + 112 + 3$
$= -56 + 115 \Rightarrow 59 \text{ ft}$

c) Evaluate *h*(0). What does this value tell you? Explain in the context of the problem.

$h(0) = -14(0^2) + 56(0) + 3$
$h(0) = 3$

$(0, 3)$
0 sec — 3 ft

THE BALL LEAVES THE BAT 3 FEET OFF THE GROUND.

d) How long is the ball in the air?

- IT IS IN THE AIR UNTIL IT HITS THE GROUND.
- SO, WHEN DOES THE BALL HIT THE GROUND? AT X-INTERCEPT.
- IF IT DOESN'T FACTOR, USE QUADRATIC FORMULA TO SOLVE: $X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$A = -14$
$B = 56$
$C = 3$

$X = \frac{-56 \pm \sqrt{56^2 - 4(-14)(3)}}{2(-14)}$

$= \frac{-56 \pm \sqrt{3136 + 168}}{-28}$

$= \frac{-56 \pm \sqrt{3304}}{-28}$

$\Rightarrow X = \frac{-56 \pm 57.48}{-28}$

$X = \frac{1.48}{-28}$ — $X = \frac{-113.48}{-28}$

$X = -0.05 \text{ SEC.}$ — $Y = 4.05 \text{ SEC.}$

Ticket out the door:

In the last problem how would have the function changed had Tim hit a line drive that the 2nd baseman caught?