AP Lunchtime Review Sessions #7
Statistical Inference: Confidence Intervals

Statistical Inference: CONFIDENCE INTERVALS & SIGNIFICANCE TESTS
- FORM CONCLUSIONS ABOUT POPULATION, USES PROBABILITY TO EXPRESS THE STRENGTH OF THE CONCLUSIONS.
- USING SAMPLING DISTRIBUTIONS OF STATISTICS - EXPLORES WHAT WOULD HAPPEN IF THE PROCESS REPEATS MANY TIMES.
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Confidence Intervals: RANGE OF PLAUSIBLE VALUES FOR A PARAMETER; CREATED USING SAMPLE DATA & SAMPLING DISTRIBUTIONS

- Calculating a confidence Interval: STATISTIC ± CRITICAL VALUE (STANDARD ERROR)

Margin of Error: difference between point estimate and the true parameter value; how "wrong" you're willing to be in estimating

- Interpreting Confidence levels

"IF WE TAKE MANY SAMPLES OF SAME SIZE FROM THE POPULATION, ABOUT C% OF THEM WILL RESULT IN AN INTERVAL THAT CAPTURES THE ACTUAL PARAMETER VALUE."

- Interpreting Confidence Intervals

WE ARE ___% CONFIDENT THAT THE INTERVAL FROM ___ TO ___ CAPTURES THE TRUE [parameter in context]...!!.

Proportions CATEGORICAL DATA	Means QUANTITATIVE DATA	
ONE SAMPLE: p = True proportion of ___ that...	ONE SAMPLE: μ = TRUE MEAN OF ___	• DEFINE PARAMETER YOU ARE ESTIMATING State
TWO SAMPLE: $p_1 - p_2$ = True difference of = proportion of ___ and ___ who...	TWO SAMPLE: $\mu_1 - \mu_2$ = TRUE DIFFERENCE IN MEANS OF ___ & ___	
One sample / 2 sample z- interval for p / $p_1 - p_2$ • Random? • 10% Large Counts all $n \cdot p \geq 10$ and $n(1-p) \geq 10$	ONE SAMPLE / 2 SAMPLE t- INTERVAL FOR μ or $\mu_1 - \mu_2$ NORMAL / LARGE SAMPLE: POPULATIONS are normal, or $n \geq 30$	• IDENTIFY THE APPROPRIATE INFERENCE METHOD Plan CHECK CONDITIONS
One sample: $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $< p <$	ONE SAMPLE: $\bar{x} \pm t^* \frac{s_x}{\sqrt{n}}$ $< \mu <$	Do CALCULATIONS
Two Sample: $(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ $< p_1 - p_2 <$	TWO SAMPLE: $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ $< \mu_1 - \mu_2 <$	
		Conclude

Chapter 8 Multiple Choice Practice

Directions. *Identify the choice that best completes the statement or answers the question. Check your answers and note your performance when you are finished.*

1. Gallup Poll interviews 1600 people. Of these, 18% say that they jog regularly. A news report adds: "The poll had a margin of error of plus or minus three percentage points." You can safely conclude that
A. 95% of all Gallup Poll samples like this one give answers within ±3% of the true population value.
B. the percent of the population who jog is certain to be between 15% and 21%.
C. 95% of the population jog between 15% and 21% of the time.
D. we can be 3% confident that the sample result is true.
E. if Gallup took many samples, 95% of them would find that 18% of the people in the sample jog.

2. An agricultural researcher plants 25 plots with a new variety of corn. A 90% confidence interval for the average yield for these plots is found to be 162.72 ± 4.47 bushels per acre. Which of the following is the correct interpretation of the interval?
A. There is a 90% chance the interval from 158.28 to 167.19 captures the true average yield.
B. 90% of sample average yields will be between 158.28 and 167.19 bushels per acre.
C. We are 90% confident the interval from 158.28 to 167.19 captures the true average yield.
D. 90% of the time, the true average yield will fall between 158.28 and 167.19.
E. We are 90% confident the true average yield is 162.72.

3. I collect a random sample of size n from a population and from the data collected compute a 95% confidence interval for the mean of the population. Which of the following would produce a wider confidence interval, based on these same data?
A. Use a larger confidence level.
B. Use a smaller confidence level.
C. Use the same confidence level, but compute the interval n times. Approximately 5% of these intervals will be larger.
D. Increase the sample size.
E. Nothing can ensure that you will get a larger interval. One can only say the chance of obtaining a larger interval is 0.05.

4. A marketing company discovered the following problems with a recent poll:
I. Some people refused to answer questions
II. People without telephones could not be in the sample
III. Some people never answered the phone in several calls
Which of these sources is included in the ±2% margin of error announced for the poll?
A. Only source I.
B. Only source II.
C. Only source III.
D. All three sources of error.
E. None of these sources of error.

5. You are told that the proportion of those who answered "yes" to a poll about internet use is 0.70, and that the standard error is 0.0459. The sample size
A. is 50.
B. is 99.
C. is 100.
D. is 200.
E. cannot be determined from the information given.

6. The standardized test scores of 16 students have mean $\bar{x} = 200$ and standard deviation $s = 20$. What is the standard error of $\bar{x}$?

A. 20
B. 10
C. 5
D. 1.25
E. 0.80

7. A newspaper conducted a statewide survey concerning the 2008 race for state senator. The newspaper took a random sample (assume it is an SRS) of 1200 registered voters and found that 620 would vote for the Republican candidate. Let p represent the proportion of registered voters in the state that would vote for the Republican candidate. A 90% confidence interval for p is

A. 0.517 ± 0.014.
B. 0.517 ± 0.022.
C. 0.517 ± 0.024.
D. 0.517 ± 0.028.
E. 0.517 ± 0.249.

8. After a college's football team once again lost a football game to the college's arch rival, the alumni association decided to conduct a survey to see if alumni were in favor of firing the coach. Let p represent the proportion of all living alumni who favor firing the coach. Which of the following is the smallest sample size needed to guarantee an estimate that's within 0.05 of p at a 95% confidence level?

A. 269
B. 385
C. 538
D. 768
E. 1436

9. An SRS of 100 postal employees found that the average time these employees had worked for the postal service was $\bar{x} = 7$ years with standard deviation $s_x = 2$ years. Assume the distribution of the time the population of employees has worked for the postal service is approximately Normal. A 95% confidence interval for the mean time μ the population of postal service employees has spent with the postal service is

A. 7 ± 2.
B. 7 ± 1.984.
C. 7 ± 0.525.
D. 7 ± 0.4.
E. 7 ± 0.2.

10. Do students tend to improve their SAT Mathematics (SAT-M) score the second time they take the test? A random sample of four students who took the test twice earned the following scores.

Student	1	2	3	4
First Score	450	520	720	600
Second Score	440	600	720	630

Assume that the change in SAT-M score (second score − first score) for the population of all students taking the test twice is approximately Normally distributed with mean μ. A 90% confidence interval for μ is

A. 25.0 ± 118.03.
B. 25.0 ± 64.29.
C. 25.0 ± 47.56.
D. 25.0 ± 43.08.
E. 25.0 ± 33.24.

Multiple Choice Answers

Problem	Answer	Concept	Right	Wrong	Simple Mistake?	Need to Study More
1	A	Interpreting Confidence				
2	C	Interpret a Confidence Interval				
3	A	Width of a Confidence Interval				
4	E	Biased Samples				
5	C	Standard Error of $\hat{p}$				
6	C	Standard Error of $\overline{x}$				
7	C	Confidence Interval for p				
8	B	Choosing Sample Size				
9	D	Confidence Interval for μ				
10	C	Confidence Interval for μ (paired data)				

FRAPPY! Free Response AP Problem, Yay!

The following problem is modeled after actual Advanced Placement Statistics free response questions. Your task is to generate a complete, concise response in 15 minutes. After you generate your response, view two example solutions and determine whether you feel they are "complete," "substantial," "developing" or "minimal." If they are not "complete," what would you suggest to the student who wrote them to increase their score? Finally, you will be provided with a rubric. Score your response and note what, if anything, you would do differently to increase your own score.

A machine at a soft-drink bottling factory is calibrated to dispense 12 ounces of cola into cans. A simple random sample of 35 cans is pulled from the line after being filled and the contents are measured. The mean content of the 35 cans is 11.92 ounces with a standard deviation of 0.085 ounce.

a) Construct and interpret a 95% confidence interval to estimate the true mean contents of the cans being filled by this machine.

b) Based on your result from a), does the machine appear to be working properly? Justify your answer.

c) Interpret the confidence level of 95 percent in context.

Student Response 1:

a) One samp-t-int = (11.89, 11.94)

b) There is a 95% chance the true mean of the amount the machine fills cans is captured in this interval.

c) If we took 100 samples, 95 of them would create an interval that captures the true mean.

How would you score this response? Is it substantial? Complete? Developing? Minimal? Is there anything this student could do to earn a better score?

Student Response 2:

a) Conditions: Random sample is given. The cans are independent of each other. Since 35>30, we can assume normality of the sampling distribution.

95% t-interval for the true mean contents:
$11.92 \pm 2.042(0.085/\sqrt{(35)}) = (11.89, 11.94)$

b) We are 95% confident the true mean contents of the cans filled by this machine falls between 11.89 and 11.94 oz. It appears the machine might be underfilling the cans since 12 oz is not in the interval.

c) If we were to take many samples of size 35 and construct intervals from their sample mean contents, 95% of the intervals would capture the true mean contents being dispensed by the filling machine.

How would you score this response? Is it substantial? Complete? Developing? Minimal? Is there anything this student could do to earn a better score?

Scoring Rubric

Use the following rubric to score your response. Each part receives a score of "Essentially Correct," "Partially Correct," or "Incorrect." When you have scored your response, reflect on your understanding of the concepts addressed in this problem. If necessary, note what you would do differently on future questions like this to increase your score.

Intent of the Question

The goal of this question is to determine your ability to construct and interpret a confidence interval and correctly interpret the confidence level in the context of a problem.

Solution

(a) Conditions: Random – The cans were randomly selected.
Independent – There are more than 10(35) cans on the line.
Normal – $n = 35$ (greater than 30), so the sampling distribution of $\bar{x}$ will be approximately normal.

95% CI for μ: $11.92 \pm 2.042(0.085/\sqrt{(35)}) = (11.89, 11.94)$

(b) We are 95% confident that the interval from 11.89 ounces to 11.94 ounces captures the true mean contents of the cans filled by this machine. It appears the machine may be filling less than it is supposed to since 12 is not in the interval.

(c) 95% of intervals constructed from random samples of 35 cans from this machine will be successful in capturing the true mean contents.

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is essentially correct if the response correctly checks the conditions for a one-sample *t* confidence interval for a mean AND correctly calculates the interval. Part (a) is partially correct if the conditions are not properly checked but the interval is correct. Note: the construction of a *z*-interval receives a partial at most.

Part (b) is essentially correct if the response correctly interprets the confidence interval in context AND correctly notes the machine appears to be underfilling because 12 is not contained in the interval. Part (b) is partially correct if the interpretation lacks context OR fails to make a decision about the machine based on the interval.

Part (c) is essentially correct if the response correctly interprets the confidence level in context. Part (c) is partially correct if the interpretation lacks context.

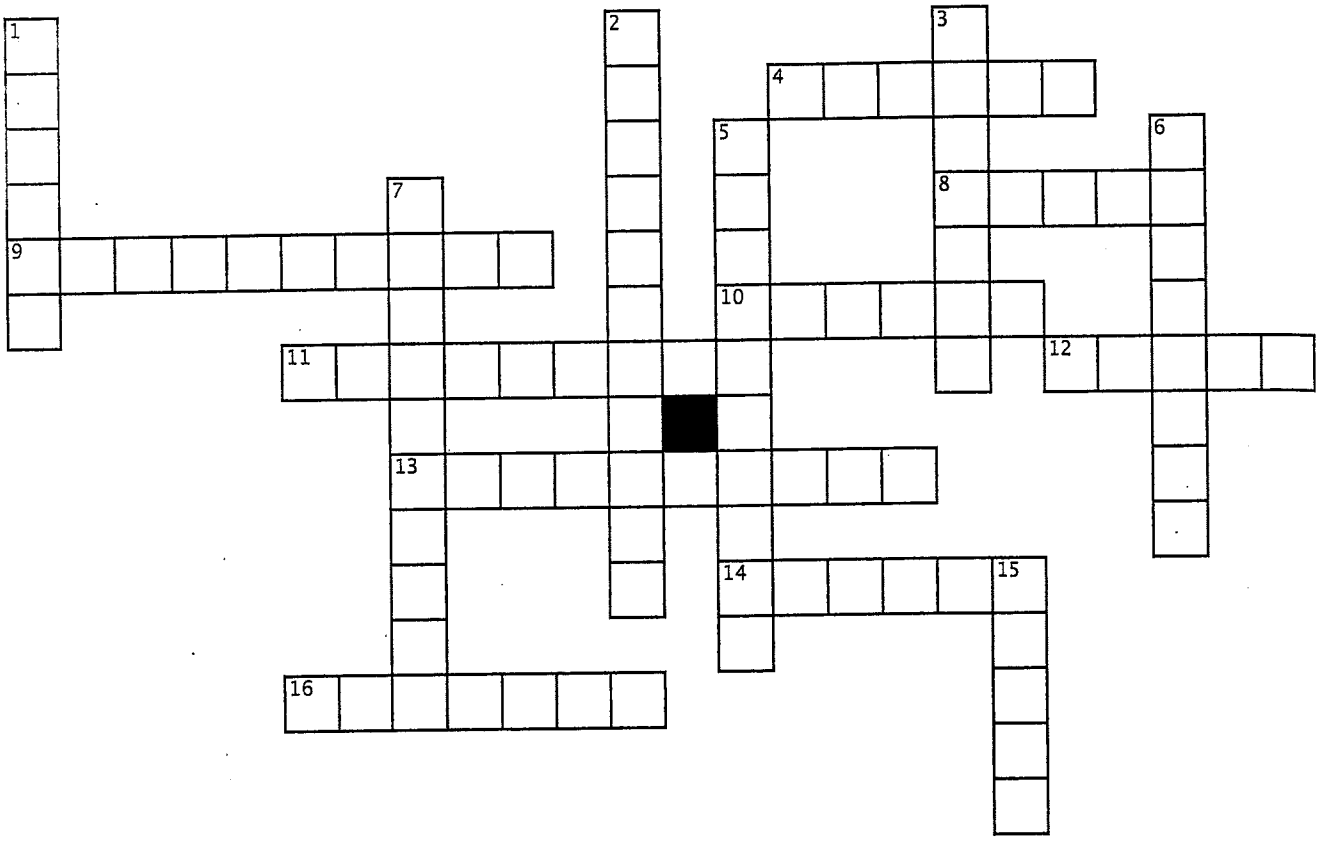
4 Complete Response
All three parts essentially correct

3 Substantial Response
Two parts essentially correct and one part partially correct

2 Developing Response
Two parts essentially correct and no parts partially correct
One part essentially correct and two parts partially correct
Three parts partially correct

1 Minimal Response
One part essentially correct and one part partially correct
One part essentially correct and no parts partially correct
No parts essentially correct and two parts partially correct

Chapter 8: Estimating with Confidence



Across

4. ____ t procedures allow us to compare the responses to two treatments in a matched pairs design
8. a confidence interval consists of an estimate ± margin of ____
9. to find the standard error of the sample mean, divide the sample standard deviation by the ____ of the sample size (two-words)
10. to estimate with confidence, our estimate should be calculated from a ___ sample
11. methods for drawing conclusions about a population from sample data
12. a single value used to estimate a parameter is a ____ estimator
13. we can construct a narrow interval by ____ our confidence
14. as degrees of freedom increase, the t distribution approaches the ____ distribution
16. the spread of the t distributions is ____ than the spread of the standard Normal distribution

Down

1. inference procedures that remain fairly accura even when a condition is violated
2. another condition for confidence intervals is th observations should be ____
3. particular t distributions are specified by degre of ____
5. we can construct a narrow confidence interval by ____ our sample size
6. the margin of error consists of a ____ value a the standard error of the sampling distribution
7. a ____ interval provides an estimate for a population parameter
15. confidence ____: the success rate of the method in repeated sampling